Conformal Prediction for Hypersonic Flight Vehicle Classification

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Background

1. We want to make a classification for the type of HFV based trajectory data.
2. The complex trajectory of change makes traditional statistical methods incompetent.
3. We apply functional data analysis tools to reduce the redundant trajectory data in order to adopt machine learning algorithms efficiently.

Initial Data

1. We simulate the HFV trajectory data based on the Dynamic models.

$$
\begin{aligned}
\dot{V} &= -D - g\sin\theta, \\
\dot{\theta} &= [L\cos\nu + (V^2/r - g)\cos\theta]/V, \\
\dot{\sigma} &= L\sin\nu/(V\cos\theta) + V\tan\phi\cos\theta\sin\sigma/r, \\
\dot{r} &= V\sin\theta, \\
\dot{\lambda} &= -V\cos\theta\sin\sigma/(r\cos\phi), \\
\dot{\phi} &= V\cos\theta\cos\sigma/r,
\end{aligned}
$$

2. We exact the trajectory data $X_n(t)$ and the corresponding labels.

$$
\begin{aligned}
P_n(t) &= \sqrt{r_n^2(t) + \lambda_n^2(t) + \phi_n^2(t)}, \\
X_n(t) &= \log(P_n(t)) - \log(P_n(0)).
\end{aligned}
$$

Initial Data

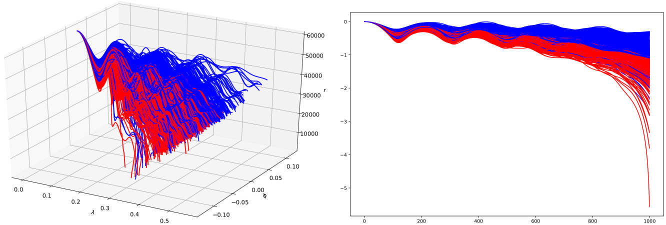


Figure: [Left] The three-dimensional trajectory plots of different maneuver models of HFV. Red lines: CAV-H (-1), blue lines: CAV-L (+1). [Right]The local-scale transform data for three-dimensional trajectory plots.

Functional Data Analysis

1. The problem of HFV trajectory classification is reduced to the problem of multiclass curves classification.
2. We apply functional data analysis tools to prepare data for machine learning algorithms.

$$X_n(t) = \sum_{m=1}^{M} c_{nm} B_m(t) = \sum_{j=1}^{p} \hat{\epsilon}_{nj} \hat{v}_j(t),$$

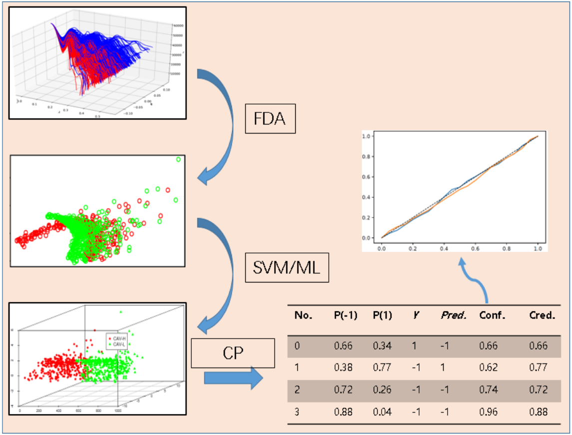
   where B_m is some standard collection of basis functions, $\hat{v}_j$ is the estimated functional principal components.
3. Once we obtain the feature attributes of HFV trajectory data,

$$(x_1, y_1), \ldots, (x_1, y_l), x \in X^p, y \in \{-1, +1\},$$

   we can adopt any machine learning algorithms efficiently.

Conformal Prediction for functional data analysis

1. We transform the trajectory data into the standard pattern recognition setting with the help of functional data analysis tools.
2. Conformal prediction can be conveniently adopt on the top of machine learning algorithms.
3. We use the inductive conformal prediction for the applications.
4. We choose *inverse probability* play the roles of nonconformity measure.
5. Both the traditional statistical methods and modern machine learning algorithms are compared.



No.	P(-1)	P(1)	*Y*	*Pred.*	Conf.	Cred.
0	0.66	0.34	1	-1	0.66	0.66
1	0.38	0.77	-1	1	0.62	0.77
2	0.72	0.26	-1	-1	0.74	0.72
3	0.88	0.04	-1	-1	0.96	0.88

Figure: The general flow of conformal prediction classification for HFV.

Table: Accuracy comparison of traditional statistical methods and modern machine learning algorithms.

Underlying algorithm	Accuracy(%)	
	B-spline	Fourier
SVMs	74.71	74.71
Decision Tree	75.37	72.05
Boosting	73.38	74.38
Neural Networks	72.55	74.04
Naïve Bayes	58.74	57.07
Logistic Regression	46.59	46.92

Table: Results of the B-spline basis function of conformal prediction for HFV.

Algorithm	1-ϵ	Accuracy(%)	=1(%)	>1(%)	$\emptyset$(%)
SVMs	99%	99.66	18.14	81.86	0.00
	95%	94.16	40.10	59.90	0.00
	90%	90.65	52.25	47.75	0.00
	80%	81.70	79.03	20.97	0.00
DT	99%	99.67	27.45	72.54	0.00
	95%	96.17	44.93	55.07	0.00
	90%	92.20	57.90	42.10	0.00
	80%	82.68	82.03	18.00	0.00
Boosting	99%	99.83	8.49	91.51	0.00
	95%	96.88	25.96	70.04	0.00
	90%	91.50	49.25	50.75	0.00
	80%	82.89	76.71	23.29	0.00
NN	99%	100	7.82	92.17	0.00
	95%	95.32	34.44	65.56	0.00
	90%	92.12	46.42	53.58	0.00
	80%	80.50	81.70	18.30	0.00
NB	99%	99.49	4.50	95.50	0.00
	95%	96.00	15.14	84.85	0.00
	90%	92.66	24.63	75.37	0.00
	80%	80.56	47.59	52.41	0.00
LR	99%	99.67	4.66	95.34	0.00
	95%	94.31	15.31	84.69	0.00
	90%	89.17	22.96	77.04	0.00
	80%	78.80	39.10	60.89	0.00

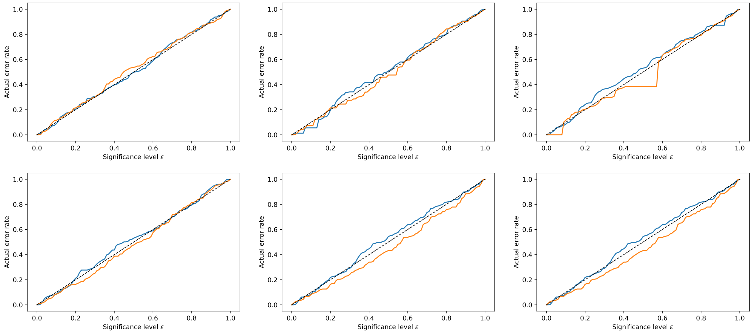


Figure: The validity check of p-values for different underlying algorithms.

Table: The example of conformal simple prediction.

#	-1	1	True Lable	CP-Label	Confidence	Credibility
0	0.872483	0.613115	-1	-1	0.386885	0.872483
1	0.845638	0.619672	-1	-1	0.380328	0.845638
2	0.184564	0.960656	1	1	0.815436	0.960656
3	0.825503	0.655738	-1	-1	0.344262	0.825503
4	0.755034	0.718033	-1	-1	0.281967	0.755034
5	0.446309	0.849180	1	1	0.553691	0.849180
6	0.580537	0.777049	1	1	0.419463	0.777049
7	0.711409	0.724590	1	1	0.288591	0.724590
8	0.963087	0.419672	-1	-1	0.580328	0.963087
9	0.436242	0.852459	1	1	0.563758	0.852459
10	0.718121	0.721311	-1	1	0.281879	0.721311
11	0.718121	0.721311	-1	1	0.281879	0.721311
12	0.718121	0.721311	-1	1	0.281879	0.721311

Table: The example of conformal set prediction.

#	-1	1	0.01	0.05	0.1	0.15	0.2	0.25	0.5	0.75	0.8	0.85	0.9	0.95	0.99	1	True Label
13	0.382550	0.773770	[-1, 1]	[-1, 1]	[-1, 1]	[-1, 1]	[-1, 1]	[-1, 1]	[1]	[1]	[]	[]	[]	[]	[]	[]	-1
14	0.721477	0.245902	[-1, 1]	[-1, 1]	[-1, 1]	[-1, 1]	[-1, 1]	[-1]	[-1]	[]	[]	[]	[]	[]	[]	[]	-1
15	0.882550	0.036066	[-1, 1]	[-1]	[-1]	[-1]	[-1]	[-1]	[-1]	[-1]	[-1]	[-1]	[]	[]	[]	[]	-1
16	0.053691	0.940984	[-1, 1]	[-1, 1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[]	[]	[]	-1
17	0.120805	0.901639	[-1, 1]	[-1, 1]	[-1, 1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[]	[]	[]	-1
18	0.087248	0.934426	[-1, 1]	[-1, 1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[]	[]	[]	-1
19	0.174497	0.875410	[-1, 1]	[-1, 1]	[-1, 1]	[-1, 1]	[1]	[1]	[1]	[1]	[1]	[1]	[]	[]	[]	[]	-1
20	0.013423	0.993443	[-1, 1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[1]	[]	1
21	0.194631	0.865574	[-1, 1]	[-1, 1]	[-1, 1]	[-1, 1]	[1]	[1]	[1]	[1]	[1]	[1]	[]	[]	[]	[]	-1

Conclusion and some critical discussion

1. It is promising for HFV classification based on trajectory data.
2. Conformal prediction work well on functional data analysis.
3. Conformal predictors are valid in simulation HFV applications.
4. Personally, it is important but unknown before I read EDBED and ALRW that SVM and machine learning algorithms are **already** functional analysis, and the FDA step is actually redundant under the view of Statistical Learning Theory.(Vapnik, 2006, pp. 447) [This work was done before I read EDBED]

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